Random Variables

1. Thus far we have considered only **finite sample spaces**, that is, sample spaces with a finite number of outcomes. In general, it is also possible to have **uncountably infinite** and **countably infinite** sample spaces. A sample space is **countable** if its outcomes can be counted using the positive integers, and **uncountable** otherwise. We will not consider uncountably infinite sample spaces in this course. However, countably infinite sample spaces can be dealt with using many of the same techniques that we've already seen.
2. Random variables
    a. Thus far, we've learned how to calculate probabilities of events, something that either does or does not happen: What is the probability that it will rain? How likely is it that I will get sick? However, there are more general questions that we can ask about experiments, questions about matters of degree: How *hard* will it rain? How *long* will this illness last? For these kinds of questions we need a new concept called **random variables**.
    b. A **random variable** is a function whose domain is the sample space (the codomain can be anything, but we'll only study random variables whose codomains are subsets of the real numbers). Note that random variables are actually functions.
    c. A special kind of random variable, called an **indicator random variable**, is a random variable whose codomain is {0, 1}. Indicator random variables are very closely related to events. An indicator random variable partions the sample space into two disjoint subsets, those outcomes that map to 1 and those that map to 0. In the same way, an event partions the sample space into two sets, those outcomes that are in the event and those that are not. Every event is naturally associated with an indicator random variable, and vice versa.
    d. There is also a strong relationship between more general random variables and events. A random variable that takes on several values partitions the sample space into several pairwise disjoint subsets. Each of these subsets is an event. Thus, we can treat an equation involving a random variable as an event.
3. Expected value of random variables
    a. The **expectation** or **expected value** of a random variable is a single descriptive number that gives us a lot of information about its behavior. Essentially, the expected value corresponds to the average value that we expect the random variable to take on. The expected value of a random variable R is given by: $E(R) = \sum_{w \in S} R(w) \cdot p(w)$.
4. Common probability distribution functions
    a. The probability distribution function for an indicator random variable is always given by a **Bernoulli distribution**:
        i. $P(X = 0) = p$
        ii. $P(X = 1) = 1 - p$

b. A random variable that takes on each of its possible values with the same probability is called **uniform**, and its probability distribution function is a **uniform distribution**.