Name: _____

**COGS Q350 Mathematical Foundations of Cognitive Science**
Review 2
To prepare for Exam 1, review and practice problems from the online text, homeworks, quizzes, handouts, and classwork.

Additional practice problems:

1. Consider the proposition $p \rightarrow (q \wedge r)$.
    a. Construct a truth table for $p \rightarrow (q \wedge r)$.

    b. Is $p \rightarrow (q \wedge r)$ a tautology, a contradiction or neither?

    c. Explain how the truth table you constructed above can be used to determine the disjunctive normal form of $p \rightarrow (q \wedge r)$.

2. Use a chain of equivalences to write the following in disjunctive normal form.

Def of → $p \to q \equiv \neg p \vee q$

a. $\neg[(s \to t) \wedge (t \to s)]$

| | |
|---|---|
| Double negation | $\neg(\neg p) \equiv p$ |
| Excluded middle | $p \vee \neg p \equiv \mathbb{T}$ |
| Contradiction | $p \wedge \neg p \equiv \mathbb{F}$ |
| Identity laws | $\mathbb{T} \wedge p \equiv p$ |
| | $\mathbb{F} \vee p \equiv p$ |
| Idempotent laws | $p \wedge p \equiv p$ |
| | $p \vee p \equiv p$ |
| Commutative laws | $p \wedge q \equiv q \wedge p$ |
| | $p \vee q \equiv q \vee p$ |
| Associative laws | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ |
| | $(p \vee q) \vee r \equiv p \vee (q \vee r)$ |
| Distributive laws | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ |
| | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ |
| DeMorgan's laws | $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$ |
| | $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$ |

b. $(\neg s \to u) \wedge (\neg s \to \neg t)$

3. Use a chain of equivalences to show the following propositions are logically equivalent.

Def of → $p \to q \equiv \neg p \vee q$

| | |
|---|---|
| Double negation | $\neg(\neg p) \equiv p$ |
| Excluded middle | $p \vee \neg p \equiv \mathbb{T}$ |
| Contradiction | $p \wedge \neg p \equiv \mathbb{F}$ |
| Identity laws | $\mathbb{T} \wedge p \equiv p$ |
| | $\mathbb{F} \vee p \equiv p$ |
| Idempotent laws | $p \wedge p \equiv p$ |
| | $p \vee p \equiv p$ |
| Commutative laws | $p \wedge q \equiv q \wedge p$ |
| | $p \vee q \equiv q \vee p$ |
| Associative laws | $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ |
| | $(p \vee q) \vee r \equiv p \vee (q \vee r)$ |
| Distributive laws | $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ |
| | $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ |
| DeMorgan's laws | $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$ |
| | $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$ |

a. $\neg p \vee (q \to r)$ and $(p \wedge q) \to r$

b. $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$.

4.

a. Give the contrapositive of:

If the sample size is too small, the experiment's results will not be useful.

b. Give the converse of:

If you create a neural network to solve the problem, you found a new approach.

5. Give formal proofs of the following.

| | |
|---|---|
| Given: | $B \rightarrow (C \rightarrow \neg E)$ |
| | $\neg A \vee B$ |
| | $\neg E \rightarrow F$ |
| | $A \rightarrow (C \wedge \neg F)$ |
| Prove: | $\neg A$ |

| | |
|---|---|
| Given: | $\neg B \rightarrow \neg C$ |
| | $(A \vee E) \rightarrow (C \wedge F)$ |
| Prove: | $A \rightarrow (B \vee D)$ |