Name: ____ (1 point)

COGS Q350 Mathematical Foundations of Cognitive Science
Review

100 points.

1. (8 points; 4 points each) Let S = We build a sentient robot, E = We must address ethical questions, T = We can treat the robot like a machine, A= We receive many awards.

   1a. Translate from English to propositional logic:

   If we build a sentient robot, we must address ethical questions, but if we do not build a sentient robot, we can treat the robot like a machine.

   1b. Translate from propositional logic to English:

   $(S \wedge A) \vee \neg(S \vee A)$

2. (8 points; 4 point each) Express the following propositions in predicate logic. Make up predicates as you need. State what each predicate means. Also state the domain of discourse for each predicate.

   2a. There is one class that all of my friends have taken.

   2b. There is exactly one Venezuelan.

3. (9 points; 3 point each) Let G(x,y) stand for “x earns a high grade in course y, where the domain of discourse for x consists of students and the domain of discourse for y consist of courses.

   Translate each of the following propositions into an unambiguous English sentence.

   3a. $\forall x \exists y G(x,y)$

   3b. $\forall y \exists x G(x,y)$

   3c. $\exists x \forall y G(x,y)$

4. (8 points) Construct a truth table to show whether the following proposition is a tautology, a contradiction or neither: $(\neg P \vee \neg Q) \leftrightarrow \neg(P \vee Q)$

   Circle the correct answer: Tautology Contradiction Neither

5. (8 points) Explain how truth tables can be used to show two propositions are logically equivalent. Consider using $P \rightarrow Q$ and $\neg P \vee Q$ as examples in your explanation.

6. (16 points; 8 points each) Use a chain of equivalences to show the following propositions are logically equivalent.

Def of $\rightarrow$ $\quad p \rightarrow q \equiv \neg p \vee q$

Double negation	$\neg(\neg p) \equiv p$
Excluded middle	$p \vee \neg p \equiv \mathbb{T}$
Contradiction	$p \wedge \neg p \equiv \mathbb{F}$
Identity laws	$\mathbb{T} \wedge p \equiv p$ $\mathbb{F} \vee p \equiv p$
Idempotent laws	$p \wedge p \equiv p$ $p \vee p \equiv p$
Commutative laws	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$
Associative laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
DeMorgan's laws	$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$ $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$

6a. $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

6b. $\neg(p \rightarrow q)$ and $p \wedge \neg q$

7. (8 points; 4 points each)

7a. Give the contrapositive of:

If you run more than 20 subjects in the experiment, you will get accurate results.

7b. Give the converse of:

If you earn 100% on this exam, you answered this question correctly.

8. (24 point; 8 points each) Give formal proofs of the following.

Given: $(A \vee B) \rightarrow C$
$\neg C \vee D$
$A \rightarrow \neg D$
Prove: $\neg A$

Given: $\neg B \rightarrow \neg C$
$A \rightarrow C$
Prove: $A \rightarrow (B \vee D)$

Given: A v B
¬C → E
A → ¬E
B → D
Prove: C v D

9. (10 points) Give an informal proof of the following argument.

Given: A → B
C v ¬B
A → ¬C
Prove: ¬A